Cellular Automata as an Environment for Simulating Electromagnetic Phenomena

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***Abstract*— In this letter, the application of cellular automata to the modeling of electromagnetic phenomena is investigated. Cellular automata are fully discrete computational models (in space, time, and variables) and are exactly computable using digital hardware. One type of cellular automata, the HPP lattice gas automaton, is applied here to the modeling of two-dimensional electromagnetic field problems. Lattice gas automata can be completely described in terms of binary variables and are capable of providing linear wave behavior. Two examples are presented to explore the proposed approach: one-dimensional plane wave propagation and plane wave scattering from a perfectly conducting rectangular cylinder.**

I. INTRODUCTION

IN THIS LETTER, we consider cellular automata as an alternative computational approach to modeling electromagnetic phenomena. The approach is a departure from the traditional differential equation description from which almost all computational electromagnetic techniques have evolved [1]. Cellular automata consist of a large, spatially discrete lattice of very simple cells that evolve in discrete time steps. The number of possible states of each cell is typically very small. The evolution of cellular automata from one state to the next is described by a deterministic rule, which is local in both space and time. For our application, the description of each cell is achieved using binary variables and continuum behavior over the lattice is obtained through local averaging of states. In the early 1980's, through experimentation with various one-dimensional cellular automata, Wolfram demonstrated that these simple systems are capable of complex dynamical behavior [2]. Cellular automata have been considered as an alternative to the traditional partial differential equation description of physical phenomena [3]–[5]. Optimal parallel architectures for the simulation of cellular systems is discussed in [6], however computational experiments or details regarding application to electromagnetic phenomena are not provided. In this letter, we present a cellular automata algorithm for modeling electromagnetic phenomena and give computational results for simple two-dimensional propagation and scattering problems.

II. THEORY

The HPP lattice gas automaton was introduced in 1976 as a conceptual model (or idealized representation) of the microscopic behavior of fluid [7]. It can be shown that the partial differential equations that model fluid dynamics (Navier-Stokes equation and the continuity equation) can be derived from various lattice gas models and that lattice gas automata yield linear wave behavior for small perturbations to an equilibrium distribution [5]. In this letter, we utilize the HPP lattice gas automaton such that its linear wave behavior can be applied to the modeling of two-dimensional TM or TE electromagnetic phenomena that can be described by a linear scalar wave equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\Phi = \frac{1}{c^2}\frac{\partial^2 \Phi}{\partial^2 t} \tag{1}$$

(assuming independence in the $\hat{z}$ direction) with the application of boundary conditions and sources.

The HPP lattice gas automaton algorithm can be described in terms of the interaction of particles on a lattice, as shown in Fig. 1. The lattice spacing is Δl in each direction. Each cell has four links, with each link representing a possible velocity state in which a particle can exist. Particles obey a boolean exclusion principle—only one particle is allowed to reside in a particular velocity state within a particular cell. The lattice gas algorithm operates in two synchronized steps. In the first step, the particles interact within cells following specific collision rules. In the subsequent step the resultant particle velocity states are transferred to adjacent cells. An example of the operation of the algorithm over a single time step Δt is illustrated in Fig. 1, where a single cell is outlined. The arrows indicate the velocity states in which the particles exist. Fig. 1(a) and (b) shows the state of the particles before and after collision, respectively. In Fig. 1(c), the particles are transferred to adjacent cells. With the HPP model only two body collisions occur, and only if the particles exist in opposing velocity states. The result is two particles existing in states at right angles to the initial pair. The outlined cell of Fig. 1 illustrates a two-body collision. For all other possible situations, including a single particle, two particles at right angles, three particles or four particles, no transformation of velocity states takes place. The transition from one state to the next is unique and can be implemented with a look-up table. With these collision and transfer rules, conservation of momentum and energy are satisfied. The lattice gas automaton is exactly computable (is not affected by finite-precision arithmetic) and is reversible.

A model of electromagnetic phenomena will incorporate polarization and allow the implementation of both Neumann and Dirichlet boundary conditions. In order to accomplish this, we use the same approach as Rothman where dual lattices

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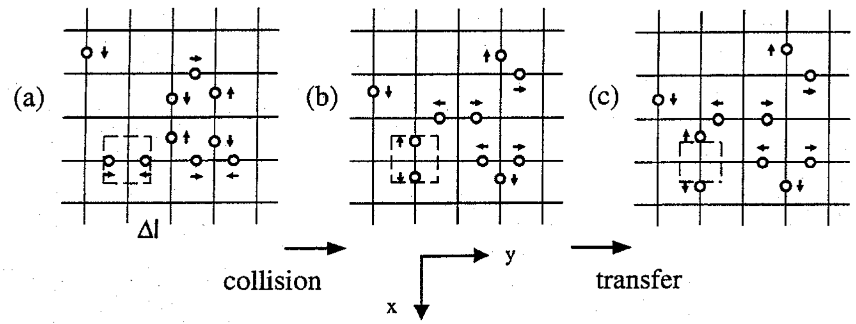


Fig. 1. Operation of the HPP lattice gas automaton over a single time step Δt. In (a), the particles are traveling toward the center of the cells. In (b) the state of the lattice is shown after the collisions have taken place, and in (c) the particles have been transferred to adjacent cells.

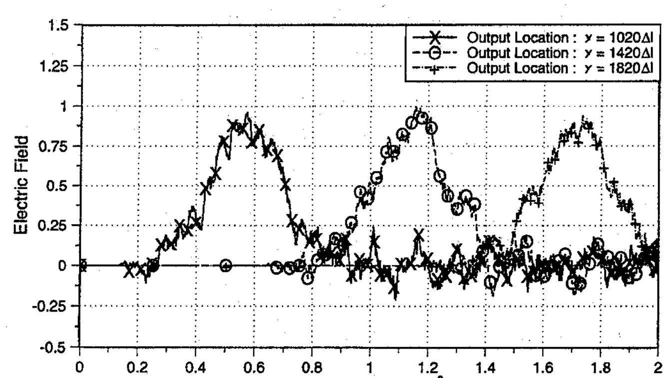


Fig. 2. Transient response at three locations of a Gaussian-pulsed plane wave propagating in a cellular automata lattice.

are employed [8]. One lattice carries *positive* particles, with the other carrying *negative* particles. Particles on the positive and negative lattices are referred to as e_+ and e_- particles, respectively. Each set of particles follow the collision rules independently, therefore the particles on the positive and negative lattices do not interact with the exception of along Dirichlet boundaries.

For the HPP automaton, the state of each cell can be described by four boolean variables, with 2^4 possible states for each cell. The microscopic state of a particular cell located in space at (x, y) can be expressed in terms of a boolean variable, $N_i(x, y)$, where i indexes the four possible velocity states (i.e., i =1 to 4). The value of $N_i(x, y)$ is 1 if a particle at the cell location (x, y) exists in velocity state i, and 0 otherwise. For a two-dimensional TM model ($\Phi \equiv E_z$), the *microscopic* electric field at each cell is defined as

$$E_{z,cell}(x, y) = \sum_{i=1}^{4} \left(N_i^+(x, y) - N_i^-(x, y)\right) \quad (2)$$

where N_i^+ and N_i^- describe the state of the cell at (x, y) within the positive and negative lattice, respectively. Macroscopic quantities are defined by local averaging of the particle distribution. For example, the *macroscopic* electric field E_z, at a particular spatial location (x, y) is defined as

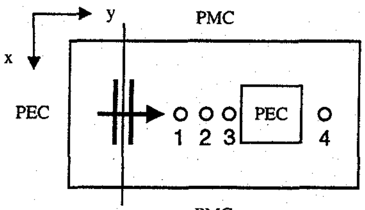
$$E_z(x, y) = \sum_R \left(\sum_{i=1}^{4} \left(N_i^+(x_R, y_R) - N_i^-(x_R, y_R)\right)\right) \quad (3)$$

where R describes a neighborhood of cells centered around (x, y), and (x_R, y_R) is the location of a particular cell within R. If a sufficiently large neighborhood (and correspondingly fine grain mesh) is used, it is possible to obtain a *continuum* approximation [3].

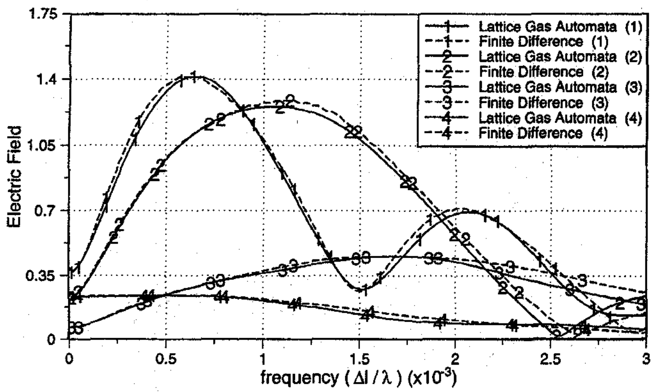
Boundary conditions for the dual HPP lattice have been developed in order to properly truncate the lattice and implement scattering obstacles. A perfect electric conducting boundary condition (PEC) is modeled by enforcing a zero microscopic tangential electric field at the boundary, (i.e., a e_+ particle incident at a PEC boundary is transferred to the e_- lattice at the following time step). A perfect magnetic conducting boundary condition (PMC) is modeled by enforcing a zero microscopic tangential magnetic field at the boundary.

III. Results

Results obtained from our lattice gas automata numerical experiments are provided to demonstrate wave propagation and scattering. In the first example considered, a Gaussian-pulsed plane wave is propagated through the lattice. The simulation space is defined as $0 < x < 600\Delta l$ and $0 < y < 2400\Delta l$. PMC boundary conditions are applied parallel to the y axis (at the top and bottom of the lattice), and PEC boundary conditions are applied parallel to the x axis (at the left and right of the lattice). A uniform background density of 0.5 (50% of all possible states are randomly filled) is applied to both the positive and negative lattices. A plane wave is excited by superimposing a Gaussian distribution (centered at $y = 620\Delta l$) of particles on top of the existing background distribution, at $t = 0$. For this example 25% of the unoccupied states above the background level in the positive lattice are filled on average at $y = 20\Delta l$. The pulse width of the Gaussian distribution is $120\Delta l$. Observation points are located along the center of the lattice ($x = 300\Delta l$) at $y = 1020$, 1420, and $1820\Delta l$. A circular sampling window of radius $30\Delta l$ is used to determine the macroscopic field E_z. In the experiment, the PEC boundary conditions are located a sufficient distance from the source and observation locations such that the simulation effectively takes place in an infinite lattice. The simulation is evolved for $2000\Delta t$. In Fig. 2, the transient response at each observation location is provided. An appropriate delay is observed between the response at each output location, indicating a propagation velocity of $c = \Delta l/(\sqrt{2}\Delta t)$. In Fig. 2, the transient signal is corrupted with a large high-



(a)



(b)

Fig. 3. Interaction of a Gaussian-pulsed plane wave with a PEC rectangular cylinder demonstrating two-dimensional TM scattering with (a) the lattice, and (b) a comparison of the frequency-difference and time-domain simulation at four observation locations within the lattice.

frequency noise. The noise is a result of the background random distribution and depends on the size of the sampling window. The noise does not significantly affect results after a discrete Fourier transform is applied.

To further examine the cellular automata approach, we consider a simple two-dimensional wave scattering problem as shown in Fig. 3. A Gaussian-pulsed plane wave source is excited at $y = 1000\Delta l$ within a lattice defined as $0 < x < 1000\Delta l$ and $0 < y < 2200\Delta l$. A perfect electrical conducting rectangular cylinder is placed within the mesh, defined by the planes $x_{\min} = 405\Delta l$, $x_{\max} = 594\Delta l$, and $y_{\min} = 1500\Delta l$, $y_{\max} = 1599\Delta l$. The top and bottom lattice boundaries are terminated with PMC boundary conditions, and the left and right lattice boundaries are terminated with PEC boundary conditions. The simulation is evolved for $1700\Delta t$. In Fig. 3, the frequency domain response obtained from the lattice gas simulation are provided for four observation locations: 1) $x = 500\Delta l$, $y = 1250\Delta l$; 2) $x = 500\Delta l$, $y = 1360\Delta l$; 3) $x = 500\Delta l$, $y = 1470\Delta l$; and 4) $x = 500\Delta l$, $y = 1739\Delta l$. Results from a finite-difference time-domain simulation are also shown for comparison and indicate reasonable agreement.

IV. Discussion

In this letter, the application of cellular automata to the modeling of electromagnetic phenomena has been considered. Cellular automata are discrete models of physical phenomena that are exactly computable using digital hardware. The HPP lattice gas automaton has been applied, which is described by binary variables with continuum behavior obtained through local averaging of the discrete states. Correct qualitative behavior and reasonably accurate quantitative results have been obtained from our version of the HPP lattice gas automaton. The cellular automata approach is not equivalent to an integer arithmetic implementation of a transmission line matrix or finite-difference time-domain algorithm. The cellular automata approach to modeling physical systems is based on the use of simple microscopic interactions from which differential equation behavior emerges on a macroscopic scale after statistical averaging. The ability to model complex phenomena, such as that due to dispersive or nonlinear media, may be possible with suitable cellular automata rules. Although the computational complexity of a cellular automata unit cell is considerably less when compared to a finite-difference time-domain approach, a much finer mesh must be used. For the example considered in Fig. 3 of this letter, the HPP cellular automaton lattice spacing was 10 times finer than that of the equivalent finite-difference time-domain mesh, so that the total number of cell updates drastically increases by a factor of 1000 (two spatial and one temporal dimension). Even with this added burden, however, since the unit cell in the lattice gas requires only a few bits of memory and simple binary operations, the technique does lend itself to very efficient implementation with special purpose computational hardware [4]–[6], [9].

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